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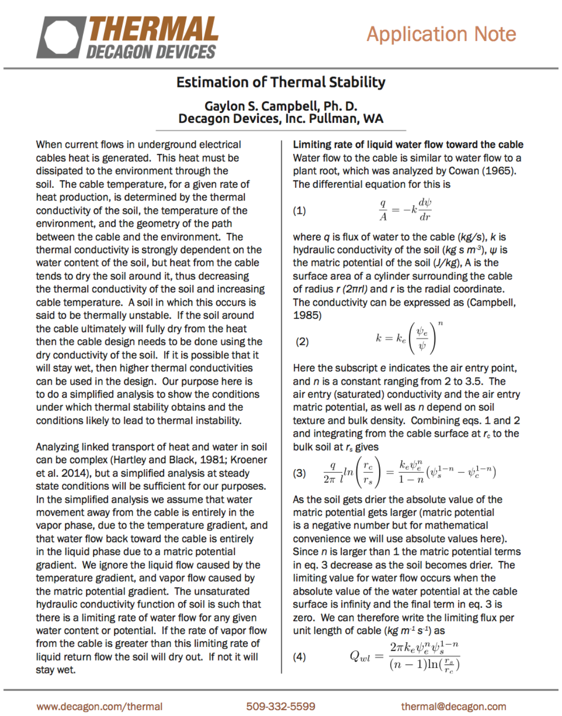
Special Notes: Illustrations are Ref Only ** Not to Scale **



THERMAL DECAGON DEVICES — Application Note

Estimation of Thermal Stability

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When current flows in underground electrical cables heat is generated. This heat must be dissipated to the environment through the soil. The cable temperature, for a given rate of heat production, is determined by the thermal conductivity of the soil, the temperature of the environment, and the geometry of the path between the cable and the environment. The thermal conductivity is strongly dependent on the water content of the soil, but heat from the cable tends to dry the soil around it, thus decreasing the thermal conductivity of the soil and increasing cable temperature. A soil in which this occurs is said to be thermally unstable. If the soil around the cable ultimately will fully dry from the heat then the cable design needs to be done using the dry conductivity of the soil. If it is possible that it will stay wet, then higher thermal conductivities can be used in the design. Our purpose here is to do a simplified analysis to show the conditions under which thermal stability obtains and the conditions likely to lead to thermal instability.

Analyzing linked transport of heat and water in soil can be complex (Hartley and Black, 1981; Kroener et al. 2014), but a simplified analysis at steady state conditions will be sufficient for our purposes. In the simplified analysis we assume that water movement away from the cable is entirely in the vapor phase, due to the temperature gradient, and that water flow back toward the cable is entirely in the liquid phase due to a matric potential gradient. We ignore the liquid flow caused by the temperature gradient, and vapor flow caused by the matric potential gradient. The unsaturated hydraulic conductivity function of soil is such that there is a limiting rate of water flow for any given water content or potential. If the rate of vapor flow from the cable is greater than this limiting rate of liquid return flow the soil will dry out. If not it will stay wet.

Limiting rate of liquid water flow toward the cable
Water flow to the cable is similar to water flow to a plant root, which was analyzed by Cowan (1965). The differential equation for this is

(1) $$\frac{q}{A} = -k\frac{d\psi}{dr}$$

where *q* is flux of water to the cable (*kg/s*), *k* is hydraulic conductivity of the soil ($kg\ s\ m^{-3}$), *ψ* is the matric potential of the soil (*J/kg*), A is the surface area of a cylinder surrounding the cable of radius *r (2πrl)* and *r* is the radial coordinate. The conductivity can be expressed as (Campbell, 1985)

(2) $$k = k_e\left(\frac{\psi_e}{\psi}\right)^n$$

Here the subscript *e* indicates the air entry point, and *n* is a constant ranging from 2 to 3.5. The air entry (saturated) conductivity and the air entry matric potential, as well as *n* depend on soil texture and bulk density. Combining eqs. 1 and 2 and integrating from the cable surface at r_c to the bulk soil at r_s gives

(3) $$\frac{q}{2\pi\ l}ln\left(\frac{r_c}{r_s}\right) = \frac{k_e\psi_e^n}{1-n}\left(\psi_s^{1-n} - \psi_c^{1-n}\right)$$

As the soil gets drier the absolute value of the matric potential gets larger (matric potential is a negative number but for mathematical convenience we will use absolute values here). Since *n* is larger than 1 the matric potential terms in eq. 3 decrease as the soil becomes drier. The limiting value for water flow occurs when the absolute value of the water potential at the cable surface is infinity and the final term in eq. 3 is zero. We can therefore write the limiting flux per unit length of cable ($kg\ m^{-1}\ s^{-1}$) as

(4) $$Q_{wl} = \frac{2\pi k_e\psi_e^n\psi_s^{1-n}}{(n-1)\ln(\frac{r_s}{r_c})}$$

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