

Document Title: <b>Description, AN, KD2 therm prop analyzer vs published stats</b>		Part # and Rev. <b>13423-01</b>	
		Release Date:	
Rev.	Description	Revision By	Date

**Production Filename:** 13423 (In Product Library)

**Path to Working Files:** DecaDoc\Application Notes\Master

**Dimensions:** 8.5 inch wide, 11 inch tall

**Material:** Paper, 92 Bright White or better, 75g/m<sup>2</sup> or heavier

**Colors:** Color Print on White

**Printer:** HP Color LaserJet 8550-PS

**Finish:** None

**Adhesive:** None

**Special Notes:** Illustrations are Ref Only \*\* Not to Scale \*\* (Shown page 1 of 4)



Application Note

**The KD2 and KD2-Pro Thermal Properties Analyzers vs. ASTM and IEEE Standards**

The measurement method and analysis used by the KD2 and KD2-Pro thermal properties analyzer to obtain thermal conductivity is based on recommendations of several published standards. While the standards differ in their details, the general methods are similar for all of them. The purpose of this note is to give the background for the method, compare the KD2 and KD2-Pro hardware and measurement procedure with those of the various standards, and provide some justification for the choices made in the KD2 and KD2-Pro designs.

The method used is generally called the transient line heat source or transient heated needle method. If heat is at a constant rate,  $q$  is applied to an infinitely long and infinitely small "line" source, the temperature response of the source over time can be described by the equation:

$$\Delta T = -\frac{q}{4\pi k} E_i\left(\frac{-r^2}{4Dt}\right) \quad (1)$$

where  $k$  is the thermal conductivity of the medium in which the line is buried,  $D$  is the thermal diffusivity of the medium,  $r$  is the distance from the line at which temperature is measured, and  $E_i$  is the exponential integral.  $E_i$  is defined in the following equation, and can be approximated by the series shown:

$$-E_i(-\alpha) = \int_0^{\infty} \frac{1}{u} \exp(-\alpha u) du \quad (2)$$

$$= -\gamma - \ln \alpha + \alpha - \alpha^2/4 + \dots$$

in which  $\gamma = 0.5772\dots$  is Euler's constant and  $\alpha = r^2/4Dt$ .

The terms beyond  $\ln \alpha$  in the series expansion of  $E_i$  become negligibly small for long times when  $r$  is small and  $D$  is large, so eq. 2 can be approximated as

$$\Delta T = -\frac{q}{4\pi k} \left[ -\gamma - \ln\left(\frac{r^2}{4Dt}\right) \right] = -\frac{q}{4\pi k} \left[ \ln t - \ln\left(\frac{r^2}{4DC_p}\right) \right] \quad (3)$$

where  $C_p = \exp(\gamma)$ . Thus, after some delay, a graph of  $\Delta T$  vs.  $\ln t$  becomes a straight line with slope equal to  $q/4\pi k$ . Since two points define a straight line,  $k$  can be computed from

$$k = \frac{q(\ln t_2 - \ln t_1)}{4\pi(\Delta T_2 - \Delta T_1)} \quad (4)$$

This approximation is used in all of the transient line source standard methods for obtaining  $k$ .

Fifty years ago, when digital computers were unavailable, and computations were done by hand, there may have been some justification for such a simplified method, but there is little justification for continuing to use simplifying assumptions which produce erroneous results if the means are readily available to do better.

Approximating the exponential integral by the logarithm is one assumption made to get to eq. 4, but it isn't the only one. Real probes are neither infinitely long nor infinitely small. In addition, the ambient temperature of the sample is never constant during a measurement; there is always some temperature drift. Fortunately, the solution to the differential equations for finite length and radius probes can be obtained. For a heated cylindrical source of radius  $a$  (m) and